2-Sketches of Deduction

Ivan Di Liberti

CatMI
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Crash course in Deductive systems

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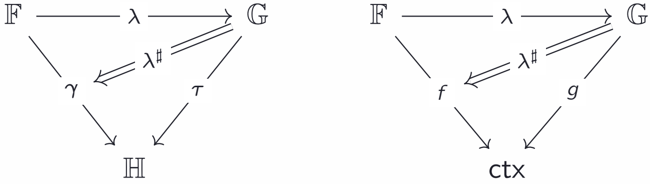

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We write

$$\Gamma \vdash A \ \ \mathbb{F} \quad iff \quad A \in f^{-1}(\Gamma)$$

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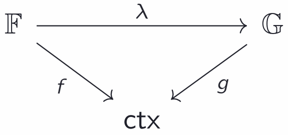

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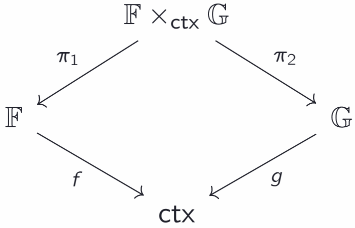

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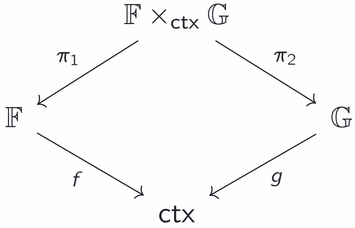

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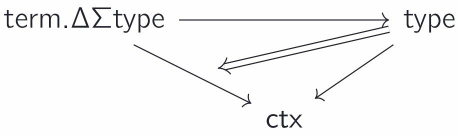

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$$Pos \simeq \mathrm{Fun}_{\mathrm{finlim}}(T, \mathrm{Set})$$

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Thm. For each of the 2-categories $\mathbb{M}$ in the previous slide, there exists a small 2-category L with finite 2-dimensional limits such that

$$\mathbb{M} \simeq \mathsf{Fun}_{\mathsf{finlim}}(L, \mathsf{Cat})$$

(in a suitable 2-dimensional sense).

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Chap 3: Final act

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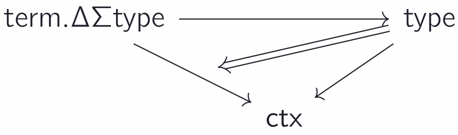

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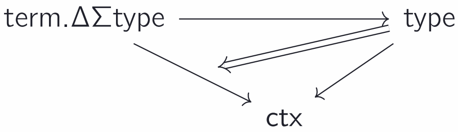

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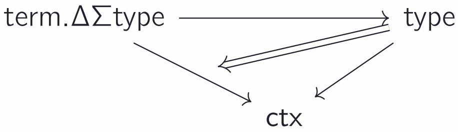

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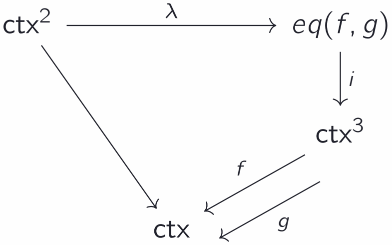

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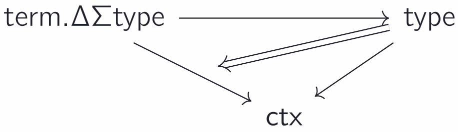

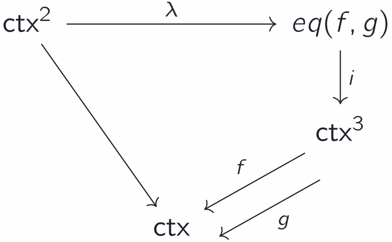

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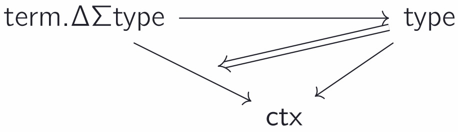

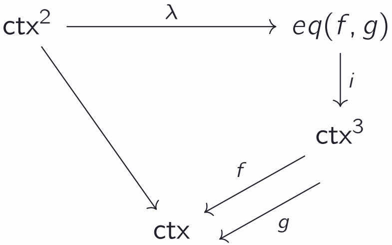

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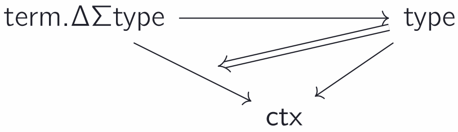

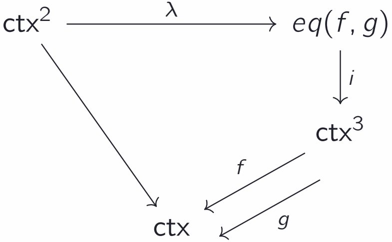

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