C A T M I 2023

Category Theory at Work in Computational Mathematics and Theoretical Informatics

June 26-30, 2023
Bergen, Norway

K. Ebrahimi-Fard (Trondheim), G. Fløystad (Bergen), H Gylterud (Bergen), H. Munthe-Kaas (Tromsø & Bergen), C. Riener (Tromsø), M. Szymik (Sheffield), U. Wolter (Bergen)

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Abstract

The meeting aims at bringing together experts from mathematics and informatics to exchange ideas on how we apply concepts and tools from category theory, type theory, and homotopy theory to structure complex problems and research in mathematics, computations and theoretical computer science.

The conference is an activity under the umbrella of the Lie–StÃÿrmer Center, a newly founded Norwegian research center for fundamental structures in computational and pure mathematics.

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Contents

1 Invited Speakers

1. Adriana Balan (Univ. Politehnica of Bucharest, Romania)
2. Nicolas Behr (CNRS, IRIF, Univ. Paris Cité, France)
3. Pierre-Louis Curien (CNRS, IRIF, Univ. Paris Cité)
4. Ivan Di Liberti (Stockholm Univ., Sweden)
5. Loïc Foissy (Univ. du Littoral Côte d'Opale, France)
6. Tobias Fritz (Univ. of Innsbruck, Austria)
7. Tomas Gonda (Univ. of Innsbruck, Austria)
8. Darij Grinberg (Drexel Univ., USA)
9. Joachim Kock (Univ. Autònoma de Barcelona & CRM, Spain)
10. Peter LeFanu Lumsdaine (Stockholm Univ., Sweden)
11. Dominique Manchon (CNRS & Univ. Clermont-Auvergne, Clermont-Ferrand)
12. Stefan Milius (Friedrich-Alexander Univ. Erlangen-Nürnberg, Germany)
13. Paige North (Utrecht Univ., Netherlands)
14. Pawel Sobocinski (Tallinn Univ. of Technology, Finland)
15. Jon Sterling (Aarhus Univ., Denmark)
16. Simon Willerton (Univ. of Sheffield, UK)
17. Fabio Zanasi (Univ. College London, UK)

2 Schedule

	Monday	Tuesday	Wednesday	Thursday	Friday
08:50 – 09:00	Opening				
09:00 – 09:45	Fritz	Sobocinski	Curien	Fritz	Balan
10:00 – 10:45	Sobocinski	Fritz	Milius	Sobocinski	LeFanu Lumsdaine
11:00 – 11:30	Coffee				
11:30 – 12:15	Kock	Foissy		North	Behr
12:30 – 13.30	Lunch				
14:00 – 14:45	Gong 1	Zanasi		Manchon	
15:00 – 15:30	Coffee		free	Coffee	end
15:30 – 16:15	Grinberg	Willerton		Di Liberti	
16:30 – 17:15	Gong 2	Sterling		Gonda	

3 Titles and Abstracts

Adriana Balan

Title: *On the tensor product of cocomplete quantale-enriched categories*

Abstract: Quantitative reasoning is often encountered in Computer Science, in various contexts and guises: in the theory of dynamical systems featuring commensurable data, such as probabilistic transition systems [14], or via behavioural distances substituting program equivalences [15], as an algebraic semantics for epistemic actions and updates [3], in Formal Concept Analysis, or as a theory of resources in quantum computing [9], and the examples could go further on. Their common feature is the semantic interpretation in domains having a metric-like structure – that is, quantales, which generalise both truth values and distances. In particular, combining actions (updates) and properties of a system [2] leads to a rich categorical structure: cocomplete quantale enriched categories [16]. In this talk we shall bring forward their symmetric tensor product [12], in a presentation featuring elements of enriched category theory, Kock–Zöberlein monads, Galois connections and Raney's G-ideals. We shall also relate nuclear objects and complete distributivity in the category of cocomplete quantale enriched categories, with an eye towards the Dedekind–MacNeille–Isbell completion.

Nicolas Behr

Title: *Double-Categorical Compositional Rewriting Theory*

Abstract: Categorical rewriting theory is a research field in computer science and applied category theory, with a rich history spanning over 50 years of active developments, starting with the pioneering work of Ehrig in the 1970s. Many of the technical developments in this field have been focussed on formalizing concepts from rewriting semantics via standard category-theoretical constructions. However, for the practically important notions of concurrency, causality or confluence, as well as for higher-level concepts of constructing continuous-time Markov chains or other types of dynamical systems based upon rewriting theories, typically each kind of categorical rewriting semantics tends to require its own variant of axiom systems to formalize the aforementioned concepts.

In this talk, I will present recent results [4] on a novel foundation for reasoning about rewriting theories via so-called compositional rewriting double categories (crDCs). The design principles followed in this approach are the typical "unify and conquer" strategy of applied category theory, and a particular variant of the notion of compositionality. Here, compositionality is introduced as a mathematical property that entails the existence of so-called Concurrency and Associativity Theorems. The former concerns being able to reason on two-step rewriting sequences via implementing the overall effect of the two-step rewrite via a composite rule applied in a single rewrite step, and vice versa. The Associativity Theorem implies that forming compositions of rewriting rules is, in a certain sense, associative. Compositionality is a necessary and crucial ingredient in the stochastic mechanics and rule algebra formalism developed by Behr et al. since 2015, and which permits to provide a mathematically fully consistent and universal formalism for continuous- [5] and discrete-time Markov Chains [6] (central to applications of rewriting to bio- and organo-chemistry, social network modeling, etc.) and rule-algebraic combinatorics [7].

References: [4, 5, 6, 7, 8]

Pierre-Louis Curien

Title: *General coherence theorems on CW-complexes and polyhedral complexes*

Abstract: We formulate and prove a coherence theorem on regular CW-complexes: 1-cells determine cellular paths, and the theorem states that any two such parallel paths (i.e. with the same end 0-cells) are provably equivalent by repetitive discrete transformations along a 2-cell if and only if each path component of the complex is simply connected. A number of coherence theorems of the literature follow as a corollary (associahedra for monoidal categories, etc.). The proof is very different from Mac Lane's original proof (which uses rewriting, even if the vocabulary of rewriting theory was not available then). We then give a second strictly less general proof of coherence, applying to polyhedral complexes satisfying a certain condition (which is in particular satisfied by all polytopes), that relies on an orientation given by some generic vector, and that retains most of the features of Mac Lane's original proof. Time permitting, we shall present a condition on polytopes of a particular kind, called nestohedra, that allows to retain "all" the spirit of Mac Lane's original proof.

Ivan Di Liberti

Title: *(2)-Sketches of deduction*

Abstract: A good framework in which to specify and compare deductive systems is crucial in order to prove general forms of existing theorems and coherently amalgamate our knowledge in mathematical logic, proof theory and type theory.

In 2021, together with Greta Coraglia we introduced the notion of judgemental theory to accommodate most of the deductive systems of the working logician. We shall start the talk by presenting this technology and quickly discuss its perks.

In 2022, with Axel Osmond, we gave a unified treatment of all the known fragments of first order logic under the general landscape of bipresentable 2-categories, whose underlying syntactic notion is that of a 2-category with finite bilimits.

After a brief presentation of the two papers, we shall amalgamate the technology of the two papers so that 2-categories with finite bilimits can offer a general framework to specify most of the known deductive systems, both from first order logic and from type theory.

Time permitting we may discuss the historical connections with the work of Power and others.

References: [10, 11]

Loïc Foissy

Title: *Parametrizations of algebraic structures*

Abstract: In the last few years, several generalizations of classical objects (associative, dendriform, pre-Lie, Rota-Baxter algebras and others) appear in the literature. In each case, each product is replaced by a family of products indexed by set (matching objects) or by a semigroup (family objects), with axioms mimicking the classical one in such a way that, loosely speaking, the underlying combinatorics is conserved.

We shall give a way to include all these generalizations in a same frame. This will make appear algebraic structures on the set used for the parametrization, such as extended (di)-associative semigroups, which are (di)-associative semigroups with extra products.

This is a joint work with Dominique Manchon, Xiao-Song Peng and Yuanyuan Zhang.

Tobias Fritz (mini-course)

Title: *Probability and Statistics with Markov Categories*

Abstract: Probability theory is traditionally based on Kolmogorov's axioms for probability measures and the concept of random variable. I will argue that this foundation is akin to programming a computer in terms of machine code, and that having a more intuitive high-level language is desirable. This leads to the formalism of Markov categories, which are an emerging framework for probability and statistics in entirely categorical terms without reference to measure theory. I will give an overview of what has been achieved so far within this formalism. This includes fully categorical treatments of theorems on sufficient statistics, the de Finetti theorem, and the law of large numbers.

Tomas Gonda (joint talk with Gemma De les Coves)

Title: *A Framework for Universality in Physics, Computer Science and Beyond*

Abstract: Universal Turing machines, universal spin models and NP-completeness are some examples of universality. While each is formulated in their own world, they all share a common structure-the property of universality per se. What is this property? And how does it manifest itself in the above and other examples? In this work, we set up a categorical framework for universality, which includes as instances the ones mentioned above and more. Given a set of targets and contexts, we call a simulator universal if it can reach all targets for all contexts. We compare simulators and distinguish trivial from non-trivial ones. We also present the relation to unreachability from Lawvere's Fixed Point Theorem. Finally, we compare instances of the framework. We also compare our framework to Pavlovic's monoidal computer. This work sets the basis for a unified approach to studies of universality and invites the instantiation of other examples. (Work with Gemma De les Coves, Sebastian Stengele, and Tobias Reinhart.)

Darij Grinberg

Title: *TBA*

Abstract: TBA

Joachim Kock

Title: *Whole-grain Petri nets and processes*

Abstract: Petri nets are a useful model for concurrency, and also serve to model processes in many other application areas in science and engineering, such as chemistry, epidemiology, production and business modelling, and so on. Their operational semantics come in two main flavours: geometric and algebraic. People have struggled for many years to reconcile the two viewpoints, the problem being an issue with symmetries. In this talk I will explain how the problem can be overcome with the help of some elementary homotopy viewpoints, and a very slight adjustment to the usual definition of Petri net. The new formalism for Petri nets, called 'whole-grain', is based on polynomial-style finite-set configurations and etale maps. The processes of a whole-grain Petri net P are etale maps G -> P from graphs. The main result I want to arrive at in the talk is that P-processes (the geometric semantics) form a symmetric monoidal Segal space, and that this is the free prop-in-groupoids on P (thus at the same time the algebraic semantics). But most of the talk will be spent just explaining Petri nets, markings, firings, and the token game, and I will also spend some time on background in simplicial homotopy theory.

Reference: [13]

Peter LeFanu Lumsdaine

Title: *Thirteen ways of looking at the cumulative hierarchy*

Abstract: In ZF(C) and similar material set-theoretic foundations, the cumulative hierarchy of iterative sets provides the arena in which all mathematics takes place. In other foundations, its place is less central; but it still exists, as a rich and useful structure.

In this mainly expository talk, I will survey various analyses of the cumulative hierarchy from non-ZF-like perspectives, drawing together strands from category theory, type theory, and Quine's non-well-founded set theory NF.

Dominique Manchon

Title: *Curvature, torsion and special polynomials*

Abstract: Considering a smooth manifold M endowed with an affine connection, special polynomials are multilinear quantities which are expressed by composing torsion, curvature and their derivatives. A long-standing conjecture stipulates that any multilinear expression which is tensorial (i.e. $C^\infty(M)$-linear with respect to each of its arguments) should be special. I will give a partial answer to this conjecture, based on a post-Lie algebra point of view on A. V. Gavrilov's algebraic approach to connections.In particular, I will provide a detailed description of the post-Lie algebra naturally associated with a manifold with connection, and explain the key role the post-Lie Magnus expansion plays in Gavrilov's double exponential. Joint work [1] with Mahdi J. H. Al-Kaabi, Kurusch Ebrahimi-Fard and Hans Munthe-Kaas.

Stefan Milius

Title: *Efficient Coalgebraic Partition Refinement (From Categorical Constructions to Tool Implementation)*

Abstract: We present a generic and efficient partition refinement algorithm that quotients coalgebraic systems by behavioural equivalence, an important task in system analysis and verification. Coalgebraic generality allows us to cover not only classical relational systems but also various forms of weighted systems and furthermore to flexibly combine existing system types. In fact, a generic complexity analysis and an implementation is enabled by assuming that the coalgebraic type functor comes equipped with a simple refinement interface, which encapsulates the functor specific parts of the algorithm and allows representing finite coalgebras in terms of nodes and edges. Instances of our algorithm match the runtime of the best known algorithms for unlabelled transition systems, Markov chains, deterministic automata (with fixed alphabets), Segala systems, and for color refinement. For weighted automata, and more generally, weighted tree automata, with weights in a cancellative monoid we still match, and for non-cancellative monoids such as (the additive monoid of) the tropical semiring we even substantially improve, the asymptotic runtime of the best known algorithms.

Time permitting we will also present the generic tool CoPaR which implements our algorithm, is easily instantiated to new concrete system types, and is modular, so that minimizers for new types of systems are obtained easily by composing pre-implemented basic functors. Experiments show that even for complex system types, the tool is able to handle systems with millions of transitions. However, memory consumption turns out to be a bottleneck. We have therefore also extended an algorithm due to Blom and Orzan, which is suitable for a distributed implementation to the coalgebraic level of genericity, and implemented it in CoPaR. Experiments on a high performance cluster show that this allows to handle much larger state spaces, while running times remain low in most experiments.

Paige North

Title: *Coinductive control of inductive data types*

Abstract: In classical programming language theory, characterizing data types as initial algebras of an endofunctor that represents a specification of the data types is an important tool. In this work, we observe that the category of algebras of such an endofunctor is often enriched in its category of coalgebras. This enrichment carries

strictly more information than the traditional, unenriched category. For example, when considering the endofunctor whose initial algebra is the natural numbers, we find that the enrichment encodes a notion of 'partial' homomorphism, while the unenriched category encodes only 'total' homomorphisms. We can also leverage this extra information to generalize the notion of initial algebras, following the theory of weighted limits.

Joint work with Maximilien Péroux: arXiv:2303.16793

Pawel Sobocinski (mini-course)

Title: *Diagrammatic relational algebra and its applications*

Abstract: Following an introduction to monoidal categories and string diagrams, the focus will be on graphical linear algebra and its extensions. This will include a summary of the basic underlying algebraic structure, which features an interaction of bialgebra and Frobenius algebra. The following additional topics will be covered:

1. string diagrammatic treatment of signal flow graphs and relational style reasoning about recurrence relations;

2. graphical affine algebra, giving a calculus for reasoning about classical non-passive electrical circuits;

3. feedback and state constructions, giving a calculus for concurrent computation, giving a semantics for models of computation such as Petri nets.

This content draws from published joint work with Filippo Bonchi, Fabio Zanasi, Robin Piedeleu, Guillaume Boisseau and others.

Jonathan Sterling

Title: *Naive Denotational Semantics: Synthetic Domains in the 21st Century*

Abstract: It is easy to teach a student how to give a naíve denotational semantics to a language like System T, and then use it to reason about the equational theory: a type might as well be a set, and a program might as well be a function, and equational adequacy at base type is established using a logical relation between the initial model and the category of sets. Adding any non-trivial feature to this language (e.g. general recursion, polymorphism, state, etc.) immediately increases the difficulty beyond the facility of a beginner: to add recursion, one must replace sets and functions with domains and continuous maps, and to accommodate polymorphism and state, one must pass to increasingly inaccessible variations on this basic picture.

The dream of the 1990s was to find a category that behaves like SET in which even general recursive and effectful programming languages could be given nav́e denotational semantics, where types are interpreted as "sets" and programs are interpreted as a "functions", without needing to check any arduous technical conditions like continuity. The benefit of this synthetic domain theory is not only that it looks "easy" for beginners, as more expert-level constructions like powerdomains or even domain

equations for recursively defined semantic worlds become simple and direct. Although there have been starts and stops, the dream of synthetic domain theory is alive and well in the 21st Century. Today's synthetic domain theory is, however, both more modular and more powerful than ever before, and has yielded significant results in programming language semantics including simple denotational semantics for an state of the art programming language with higher-order polymorphism, dependent types, recursive types, general reference types, and first-class module packages that can be stored in the heap.

In this talk, I will explain some important classical results in synthetic domain theory as well as more recent results that that illustrate the potential impact of "naí denotational semantics" on the life of a workaday computer scientist.

Simon Willerton

Title: *Metric-like spaces as enriched categories*

Abstract: A metric space is a set of points such that between each pair of points there is a number - the distance - such that the triangle inequality is satisfied; a small category is a set of objects such that between each pair of objects there is a set - the hom-set - such that elements of the hom-sets can be composed. The analogy between the structures that can be made in to a common generalization of the two structures, so that both are examples of enriched categories. This gives a bridge between category theory and metric space theory. I will describe this and give examples from around mathematics where this has perspective has been useful or interesting.

Fabio Zanasi

Title: *Categorical Semantics of Learning by Gradient Descent*

Abstract: I will present recent and ongoing work on giving a semantic foundation to training algorithms in machine learning using the categorical formalisms of lenses. Lenses provide a much needed unifying perspective on various classes of such algorithms, as well as offering a different style of specifying and proving properties of training protocols. They also enable the study of machine learning for new classes of models such as Boolean circuits and polynomial circuits. In the last part of the talk I will discuss some applications and directions for future work, including circuit design for hardware implementation, formal verification of learning algorithms, and accounting for learning beyond stochastic gradient descent.

4 Wednesday afternoon activity

We propose two options (*more details soon at www.catmi.no*):

1. Trip to Troldhaugen, including a concert.

   Lunch at Troldhaugen; It is also possible to swim there if the weather is nice.

2. Boat trip: Fjord cruise,

   Lunch at the hotel; Boat trip starts at 14.00.

5 Participants

1. Fredrik Bakke, NTNU
2. Elisabeth Bonnevier, UiB
3. Gecia Bravo-Hermsdorff, University College London
4. Morten Brun, UiB
5. Diego Caudillo, NTNU
6. Leihao Chen, University of Amsterdam
7. Subbaro Venkatesh Guggilam, NTNU
8. Lee M. Gunderson, University Colleg London
9. Cameron Michie, Queen Mary University of London
10. Silviu-George Pantelimon, University Politehnica of Bucharest
11. Yan Passeniouk, UiB
12. Tobias Reinhart, University Innsbruck
13. Jonatan Stava, UiB
14. Yannic Vargas, TU Graz
15. Jon Eivind Vatne, BI business school

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